THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY IN NEW YORK.

THE one hundred and ninety-eighth regular meeting of the Society was held in New York City on Saturday, April 27, 1918, extending through the usual morning and afternoon sessions. The attendance included the following thirty-three members:

Professor R. C. Archibald, Dr. F. W. Beal, Professor F. N. Cole, Professor Elizabeth B. Cowley, Professor Louise D. Cummings, Professor D. R. Curtiss, Professor L. P. Eisenhart, Dr. C. A. Fischer, Professor T. S. Fiske, Professor W. B. Fite, Professor O. E. Glenn, Dr. Olive C. Hazlett, Dr. T. R. Hollcroft, Professor Dunham Jackson, Mr. S. A. Joffe, Professor Edward Kasner, Professor C. J. Keyser, Dr. J. R. Kline, Professor P. H. Linehan, Professor R. B. McClenon, Professor L. C. Mathewson, Professor R. L. Moore, Professor F. M. Morgan, Professor Anna J. Pell, Dr. J. F. Ritt, Dr. Caroline E. Seely, Professor D. E. Smith, Professor P. F. Smith, Professor H. D. Thompson, Mr. H. S. Vandiver, Professor E. B. Van Vleck, Professor H. S. White, Mr. J. K. Whittemore.

Ex-President H. S. White presided at the morning session and Professor W. B. Fite at the afternoon session. The Council announced the election of the following persons to membership in the Society: Mr. O. S. Adams, U. S. Coast Survey; Professor W. P. Parker, Union Christian College, Pyeng Yang, Korea; Dr. E. F. Simonds, University of Illinois. Seven applications for membership in the Society were received.

Professor P. F. Smith was reelected a member of the Editorial Committee of the *Transactions*. Committees were appointed to consider the question of the publication of the recent Chicago Symposium, and to submit to the Council at the October meeting a list of nominations of officers to be elected at the annual meeting.

The following papers were read at the April meeting:

(1) Professor ARNOLD EMCH: "On plane algebraic curves with a given system of foci."

(2) Dr. J. F. RITT: "On the iteration of polynomials."

(3) Professor F. F. DECKER: "On the order of the system of equations arising from the vanishing of determinants of a given matrix."

(4) Professor O. E. GLENN: "Modular concomitant scales, with a fundamental system of formal covariants, modulo 3, of the binary quadratic."

(5) Professor J. E. ROWE: "The quinquesecant line invariant of the rational sextic curve in space."

(6) Professor F. H. SAFFORD: "Parametric equations of the path of a projectile when the air resistance varies as the nth power of the velocity."

(7) Professor C. L. E. MOORE: "Surfaces of rotation in space of four dimensions."

(8) Professor C. L. E. MOORE: "Translation surfaces in hyperspace."

(9) Dr. MARY F. CURTIS: "Note on the rectifiability of a space cubic."

(10) Professors F. R. SHARPE and VIRGIL SNYDER: "Certain types of involutorial space transformations."

(11) Dr. CAROLINE E. SEELY: "On kernels of positive type."

(12) Lieut. J. W. HOPKINS: "Some convergent developments associated with irregular boundary conditions."

(13) Dr. J. R. KLINE: "A necessary and sufficient condition that a closed connected point set that divides the plane into two domains be a simple curve."

(14) Professor EDWARD KASNER: "Equilong symmetries and a related group."

(15) Professor H. B. PHILLIPS: "Functions of matrices."

(16) Mr. G. H. HALLETT, JR.: "Linear order in three-dimensional euclidean and double elliptic spaces."

(17) Mr. H. S. VANDIVER: "On transformations of the Kummer criteria in connection with Fermat's last theorem."

(18) Mr. H. S. VANDIVER: "A property of cyclotomic integers and its relation to Fermat's last theorem (third paper)."

(19) Mr. H. S. VANDIVER: "On the first factor of the class number of a cyclotomic field."

(20) Mr. H. S. VANDIVER: "Proof of a property of the norm of a cyclotomic integer."

Mr. Hopkins's paper was communicated to the Society and read by Professor Dunham Jackson. Mr. Hallett was introduced by Professor R. L. Moore. In the absence of the authors, the papers of Professor Emch, Professor Decker,

Professor Rowe, Professor Safford, Professor C. L. E. Moore, Dr. Curtis, Professors Sharpe and Snyder, Professor Phillips, and the last two papers of Mr. Vandiver were read by title.

Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. The methods of Plücker, Siebeck, and others for determining the foci of given algebraic curves are well known. In this paper Professor Emch assumes conversely a given set of foci and determines the system of algebraic curves with the given set of foci. As an application of the method, among other problems, the equation

$$X^{n-2k}\, y^{2k} = (-1)^{n-2k} \frac{(2k)^{2k}}{n^n\, (n-2k)^{2k-n}}$$

of a special class of n-ics, with the nth roots of unity interpreted in the cartesian plane as foci, is established.

2. Let $f(x)$ be a polynomial of degree n greater than unity, and $f_p(x)$ its pth iterate. If $|x|$ is sufficiently large, $f_p(x)$ goes to infinity with p. Dr. Ritt studies the domain which is drawn to infinity. This study is made by means of a function $\varphi(x)$, introduced by L. Boettcher, which is defined by the relation

$$\varphi(x) = \lim_{p=\infty} [f_p(x)]^{1/n^p}.$$

It follows from the existence of this function that the domain drawn to infinity is a connected region. If none of the roots of $f'(x) = 0$ lies in this domain, the domain is simply connected. If the coefficient of x^n in $f(x)$ is unity, this simply connected domain covers a portion of the interior of the circle $|x| = 1$. If a root of $f'(x) = 0$ lies in the domain drawn to infinity, that domain is infinitely connected.

3. In a paper previously presented to the Society Professor Decker gave a proof of a theorem announced by Salmon without proof: The number of solutions of the system of equations arising from the vanishing of all determinants of the mth order that can be formed from a matrix with m rows and n columns, $m \geqslant n$, by the suppression of $n - m$ columns, the element in the ith row and jth column being of degree $\alpha_i + \alpha_j$

in $n - m + 1$ non-homogeneous variables, is K_{n-m+1}, where

$$K_l = \sum_{i=0}^{i=l} c_i \, \delta_{l-i},$$

c_l representing the sum all possible products of l different α's and δ_l the sum of all possible products of l α's, repetitions being permissible.

In the present paper the system arising from the vanishing of all determinants of the rth order that can be formed by the suppression of $n - r$ columns and $m - r$ rows, $r \geqslant m$, $r \geqslant n$, is treated. The order is shown to be

$$\begin{vmatrix} K_{n-r+1} & K_{n-r} & \cdots & K_{n-m+1} \\ K_{n-r+2} & K_{n-r+1} & \cdots & K_{n-m+2} \\ \cdots & \cdots & \cdots & \cdots \\ K_{n-2r+m+1} & K_{n-2r+m} & \cdots & K_{n-r+1} \end{vmatrix}.$$

If the degree of every element is b, the order reduces to

$$\frac{\prod_{i=0}^{i=m-r} {}_{n+i}C_{n-r+1} \cdot b^{(m-r+1)(n-r+1)}}{\prod_{j=1}^{j=m-r+1} {}_{n-r+j}C_{n-r+1}},$$

a result established by Segre.

4. Professor Glenn's paper is devoted primarily to a principle in modular invariant theory based, in a general theoretical way, upon analogy with the algebraical principle of symbolical convolution, by which, in both theories, an assigned covariant of a given degree becomes a member of a finite scale of derived concomitants of the same degree. Upon this principle the main theorems in algebraical invariant theory have been made to rest, and, in the present paper, the elements of a similar general modular theory are considered.

The methods developed are applied in deriving a complete system of formal concomitants of the binary quadratic form under the total binary group modulo 3.

The paper will be offered for publication in the *Transactions*.

5. In Professor Rowe's paper the $u_t^6 = 0$, $v_t^6 = 0$, and $w_t^6 = 0$ are three linearly independent binary sextics of the

double infinity of binary sextics to which all plane sections of the R_3^6, the rational sextic curve in ordinary space, are apolar, and $a_t^5 = 0$ is a binary quintic. The necessary and sufficient condition that the R_3^6 have a quinquesecant line is that $|\alpha u|^5 u_t = 0$, $|\alpha v|^5 v_t = 0$, and $|\alpha w|^5 w_t = 0$ hold for all values of t. This gives rise to an invariant of degree two in the four-rowed determinants of the matrix of the coefficients of the parametric equations of R_3^6. By a simple substitution this invariant may be transformed into an invariant of the rational plane sextic.

6. Professor Safford has obtained in parametric form the equations of the path of a projectile when the air resistance varies as the nth power of the velocity of the projectile. This problem was proposed by a member of the National Research Council and it was stated that the solution was known only for the case of $n = 2$, as given by de Sparre in the *Comptes Rendus*, volume 160. In that case certain approximations were made in the early stages of the discussion. In the present paper the accuracy of the results depends only upon the accuracy with which the physical constants can be determined.

7. Professor Moore defines a surface of revolution as one left invariant by a rotation and discusses the forms of such surfaces if the rotation is restricted to the type having an infinite number of invariant planes.

8. In this paper Professor Moore shows that no translation surfaces in hyperspace, except cylinders, can be expressed in more than one way as translation surfaces. The condition that the surface be developable is also discussed.

9. A necessary and sufficient condition that the space cubic

$$x_1 = at, \qquad x_2 = bt^2, \qquad x_3 = ct^3 \qquad (abc \neq 0)$$

be algebraically rectifiable is, as Dr. Curtis shows, that the cubic be a helix.

10. A (2, 1) correspondence between the points of two spaces (x) and (x') may be defined by three equations which are linear in (x'). The transformation which interchanges the two points in (x) which correspond to any point in (x') is

involutorial. By this method, Professors Sharpe and Snyder discuss various involutorial transformations, in particular the case in which the equations represent quartic surfaces through an octic curve of genus two. Each quartic surface is invariant under the transformation which is of order 31, and a similar transformation. The product of the two involutions is of infinite order, so that the surfaces are similar to but distinct from the known Fano quartic surfaces through a sextic of genus two.

11. Dr. Seely considers kernels that have an infinite number of iterated kernels of positive type with respect to all functions of the form $\alpha\varphi_i(s) + \beta\varphi_j(s)$ and $\alpha\psi_i(s) + \beta\psi_j(s)$, where the φ's and ψ's are characteristic functions, and shows that such kernels have certain properties in common with symmetric kernels.

12. Birkhoff (*Transactions*, 1908) has given a general theory of the convergence of the development of an arbitrary function $f(x)$ in series of characteristic solutions of an ordinary linear differential equation of the nth order with "regular" boundary conditions. It was found by Jackson (*Proceedings of the American Academy*, 1915) that, for an extensive class of irregular boundary conditions, hypotheses similar to those imposed by Birkhoff on $f(x)$ are very far from being sufficient to insure convergence of the corresponding expansion. Lieutenant Hopkins examines more carefully the case of the differential equation $d^2u/dx^2 + \rho^2 u = 0$, with the irregular boundary conditions $u(0) = u'(0) = u(\pi) = 0$. He finds that if a function $f(x)$ is to have a uniformly convergent development in terms of the characteristic solutions of this system, it must be analytic throughout a triangular region surrounding the origin in the complex x-plane, and must furthermore be of the particular form $x^2\varphi(x^3)$, where φ is an analytic function of its argument. That is, $f''(x)$ must be an analytic function of x, invariant under the transformation $x' = \omega x$, where ω is a complex cube root of unity. He proves conversely that if $f(x)$ does have the properties indicated, throughout a suitable region, its development will converge uniformly to the value $f(x)$ throughout any interval contained in $(0, \pi)$ and not reaching out to the terminal point $x = \pi$. He has thereby made an essential contribution to the theory of a class of series

which have some of the properties of power series on the one hand and of ordinary trigonometric series on the other, and which have further novel and striking properties of their own.

13. Dr. Kline proves the following theorem:

A necessary and sufficient condition that a closed, connected, connected in kleinem, plane point set should be a simple curve is that it divide its plane into two mutually exclusive domains.

14. The comparison of conformal geometry and equilong geometry brings out many fundamental differences as well as many analogies. In the present paper Professor Kasner defines the particular equilong transformations which play the same rôle in the equilong theory as Schwarzian reflections or conformal symmetries in the theory of functions of a complex variable. The analytic representation as well as the geometric interpretation is much simpler in the equilong theory. The group generated by symmetries is also obtained.

15. In the paper of Professor Phillips a canonical form is given for a function of two or more commutative matrices, analogous to Sylvester's formula for a function of a single matrix. This is used to represent analytic functions of two or more matrices. If z is a complex variable and Z and A matrices, it is shown that $f(Z)$ can be expanded in a Taylor's series in powers of $Z-A$, the series converging if each root of Z lies within a circle, with center at the corresponding root of A, in which $f(z)$ is analytic. A simple example is given of two commutative matrices not expressible as polynomials in the same matrix.

16. Mr. Hallett gives four pairs of independent categorical postulate sets for euclidean and double elliptic geometries of three dimensions in terms of point and order. Each pair has a large common basis. Each set is such that all the properties of a line can be deduced from it without the use of any postulates which necessitate the existence of two dimensions. In three pairs of sets a modification of order due to Kempe is used to advantage. A fifth treatment, involving fewer postulates than the others, is given for double elliptic geometry. The categoricity of the sets is proved by establishing the equivalence of sets of postulates given previously by Professor Veblen and Dr. Kline.

17. In an article in the Berlin *Sitzungsberichte*, June, 1914, Frobenius proved that if

$$x^p + y^p + z^p = 0 \tag{1}$$

is satisfied in integers prime to the prime p, then $11^{p-1} \equiv 17^{p-1} \equiv 1 \pmod{p^2}$ and if $p \equiv 2 \pmod 3$ also $7^{p-1} \equiv 13^{p-1} \equiv 19^{p-1} \equiv 1 \pmod{p^2}$. He also stated (loc. cit. page 670) that it is impossible to derive the congruence $23^{p-1} \equiv 1 \pmod{p^2}$ as a criterion for (1) by the use of his methods for the case $p \equiv 1 \pmod 3$ because of the appearance of a factor $t^2 - t + 1$ in the process of eliminating certain functions t. This statement of Frobenius is incorrect. In the present paper Mr. Vandiver derives the condition $23^{p-1} \equiv 1 \pmod{p^2}$ for the solution of (1) in integers prime to p, provided p is not of the form 1 (mod 11), by using only relations given by Frobenius in the article cited above.

18. In previous papers presented to the Society under this title in January and April, 1915, Mr. Vandiver derived a certain relation involving cyclotomic integers which serves as a starting point for the derivation of a number of criteria relating to Fermat's last theorem. In the present paper the criteria $B_s \equiv 0 \pmod{p^2}$

$$s = \frac{(p - 2i)\,p + 1}{2} \qquad (i = 2, 3, 4, 5),$$

for the solution of equation (1) of the preceding abstract in integers prime to p are obtained, the B's being numbers of Bernoulli, $B_1 = 1/6$, $B_2 = 1/30$, etc. These criteria reduce, modulo p, to the conditions

$$B_{\frac{1}{2}(p-3)} \equiv B_{\frac{1}{2}(p-5)} \equiv B_{\frac{1}{2}(p-7)} \equiv B_{\frac{1}{2}(p-9)} \equiv 0 \pmod p$$

given by Kummer and Mirimanoff.

19. The number of classes of ideals in a cyclotomic field defined by $l^{2i\pi/p}$, where p is prime, may be expressed as the product of two integral factors, one of which (generally known as the first factor) is as follows:

$$h = \frac{f(Z)\,f(Z^2)\,\ldots\,f(Z^{p-2})}{(2p)^{\frac{1}{2}(p-3)}}$$

where

$$f(x) = r_0 + r_1 x + \ldots + r_{p-2} x^{p-2},$$

$Z = e^{2i\pi/(p-1)}$, and r_i is the least positive residue of r^i, modulo p. Mr. Vandiver derives necessary and sufficient conditions that h be divisible by p^n, in terms of Bernoulli numbers, the argument used being different from those employed by Kummer and Kronecker in their treatment of the case $n = 1$.

20. In his first proof of the law of reciprocity between two ideals in a regular cyclotomic field Kummer gave the relation*

$$\left[\frac{d^{p-1} \log \omega(e^v)}{dv^{p-1}}\right]_{v=0} \equiv \frac{(\omega(1))^{p-1} - N(\omega)}{p} \pmod{p},$$

where

$$\omega(x) = a_0 + a_1 x + \ldots + a_{p-2} x^{p-2},$$

$$\omega = a_0 + a_1 \alpha + \ldots + a_{p-2} \alpha^{p-2},$$

$\alpha = e^{2i\pi/p}$ and $a_0, a_1, \ldots, a_{p-2}$ are integers, ω being prime to the prime p. In the present paper Mr. Vandiver gives a proof of this result which differs in character from that of Kummer.

F. N. COLE,
Secretary.

NOTE ON ISOGENOUS COMPLEX FUNCTIONS OF CURVES.

BY PROFESSOR W. C. GRAUSTEIN.

(Read before the American Mathematical Society September 5, 1917.)

LET L be a continuous, closed and directed space curve, without multiple points, let $-L$ be the same curve oppositely directed, and let $F|[L]|$ be a function of L, such that $F|[-L]| = -F|[L]|$. Form the ratio

$$\frac{F|[L']| - F|[L]|}{\sigma},$$

where L' is the directed curve obtained from L by replacing an arc PP' of L by a second arc joining P with P', and σ

* *Abhandlungen, Berlin Academy*, 1859, p. 119, formula (7).

is the area of a surface S bounded by the two arcs PP'. The limit of this ratio as the surface S shrinks to a point in P, if it exists, is called the derivative of $F|[L]|$ with respect to S in the point P and is denoted by $dF/d\sigma$. This derivative, dependent in general on L as well as on P and S, becomes for a given S a function of P alone, in the case that $F|[L]|$ is a so-called function of the first degree, that is, is a function such that the sum of its values for two directed curves, possessing a common arc oppositely directed for the two, is equal to its value for the curve resulting from the combination of the parts of the two curves other than their common arc. In this case the above ratio becomes $F|[L']|/\sigma$, where L' is the directed boundary of S, made up of the directed arcs PP' of L' and $-L$. Furthermore, in this case,

$$F|[L']| = \int_S\int (p\cos nx + q\cos ny + r\cos nz)d\sigma,$$

where p, q, r are continuous point functions and $\cos nx$, $\cos ny$, $\cos nz$ are the direction cosines of the normal to S suitably directed. Then the value of the derivative of F with respect to S in the point P is

$$\frac{dF}{d\sigma} = \lim_{\sigma=0}\frac{F|[L']|}{\sigma} = p\cos nx + q\cos ny + r\cos nz,$$

provided that the values of p, q, r, $\cos nx$, $\cos ny$, $\cos nz$ are taken in P.*

The functions p, q, r are termed the derivatives of F in P with respect to the three coordinate planes. Consider them as the components of a vector point function or vector c, and call this vector c the gradient of the function F in P. If the gradient c, besides being continuous, possesses continuous first partial derivatives, it follows, by application of the theorem of Gauss expressing a volume integral in terms of a surface integral, that

$$\text{div } c = \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y} + \frac{\partial r}{\partial z} = 0.$$

* For a more detailed development of the contents of this and the two following paragraphs, see Volterra, "Sur une généralisation de la théorie des fonctions d'une variable imaginaire," *Acta Mathematica*, vol. 12 (1889), pp. 233–286; Lévy, "Sur les equations intégro-differentielles définissant des fonctions de lignes," Doctor's Thesis, Paris, 1911, Introduction; Evans, "Topics from the theory and applications of functionals, including integral equations," The Cambridge Colloquium Lectures, delivered September 6–8, 1916, Lectures I, II.

Conversely, if div $c = 0$, where c is a vector continuous with continuous first partial derivatives, there exists a function of curves, $F\,|\,[L]\,|$, of which c is the gradient.

Consider now

$$F = F_1 + iF_2, \quad \Phi = \Phi_1 + i\Phi_2,$$

where

$$F_1 = F_1\,|\,[L]\,|, \quad F_2 = F_2\,|\,[L]\,|, \quad \Phi_1 = \Phi_1\,|\,[L]\,|, \quad \Phi_2 = \Phi_2\,|\,[L]\,|$$

are functions of L of the first degree possessing gradients c_1, c_2, γ_1, γ_2, which are continuous with continuous first partial derivatives. Form the ratio $\Delta\Phi/\Delta F$, where ΔF and $\Delta\Phi$ are the variations in F and Φ due to replacing the arc PP' of L by a second arc joining P with P'. Evidently the limit, $d\Phi/dF$, of this ratio, as the surface S bounded by the two arcs PP' shrinks to a point in P, is $(d\Phi/d\sigma) : (dF/d\sigma)$. It depends, in general, not merely on P but on the aspect of the surface S in P, namely, on the values of $\cos nx$, $\cos ny$, $\cos nz$ in P. The condition necessary and sufficient that it be independent of S is that the vectors $c_1 + ic_2$, $\gamma_1 + i\gamma_2$ be proportional, or, in other words, that the vector product $\overline{c_1 + ic_2} \times \overline{\gamma_1 + i\gamma_2}$ vanish, that is, that

$$(1) \quad c_1 \times \gamma_1 - c_2 \times \gamma_2 = 0, \quad c_1 \times \gamma_2 + c_2 \times \gamma_1 = 0.$$

If these conditions (1) are satisfied, $d\Phi/dF$ is dependent only on P, and the two complex functions $F = F_1 + iF_2$, $\Phi = \Phi_1 + i\Phi_2$ are said to be isogenous.

Lévy has noted,* without going into the details of hypothesis or proof, the general fact that for a given complex function $F = F_1 + iF_2$ there exist complex functions $\Phi = \Phi_1 + i\Phi_2$ isogenous to it, depending on two arbitrary functions of two independent parameters. In proving this, we assume that the functions F_1, F_2 possess gradients c_1, c_2, which are analytic, in general, in the coordinates x, y, z of the point P. If, then, we can establish the existence of analytic vectors γ_1, γ_2, satisfying the equations (1) and having divergences equal to zero, the existence of functions $\Phi = \Phi_1 + i\Phi_2$ isogenous to $F = F_1 + iF_2$ will follow.

The vectors γ_1, γ_2 will satisfy (1) when and only when they are linear combinations of the vectors c_1, c_2 of the form

* Loc. cit., p. 11.

$$\gamma_1 = k_1 c_1 + k_2 c_2, \quad \gamma_2 = k_2 c_1 - k_1 c_2,$$

where k_1 and k_2 are scalar point functions. Thus our problem reduces to the determination of k_1, k_2 as solutions of the equations

$$\text{div}\,(k_1 c_1 + k_2 c_2) = 0, \quad \text{div}\,(k_2 c_1 - k_1 c_2) = 0,$$

or better, as solutions of the equations (2), to which these reduce by virtue of the vanishing of the divergences of c_1, c_2,

$$\begin{aligned} &p_1 \frac{\partial k_1}{\partial x} + q_1 \frac{\partial k_1}{\partial y} + r_1 \frac{\partial k_1}{\partial z} + p_2 \frac{\partial k_2}{\partial x} + q_2 \frac{\partial k_2}{\partial y} + r_2 \frac{\partial k_2}{\partial z} = 0, \\ &p_2 \frac{\partial k_1}{\partial x} + q_2 \frac{\partial k_1}{\partial y} + r_2 \frac{\partial k_1}{\partial z} - p_1 \frac{\partial k_2}{\partial x} - q_1 \frac{\partial k_2}{\partial y} - r_1 \frac{\partial k_2}{\partial z} = 0. \end{aligned} \tag{2}$$

But according to the theory of partial differential equations there exist simultaneous solutions k_1, k_2 of the equations (2) which are analytic in general and depend on two arbitrary functions of two independent variables.

To go more into detail concerning these solutions, let us consider instead of the equations (2) the equivalent equations (3), obtained by solving the equations (2) for $\partial k_1/\partial z$, $\partial k_2/\partial z$,

$$\begin{aligned} &(r_1^2 + r_2^2)\frac{\partial k_1}{\partial z} + (p_1 r_1 + p_2 r_2)\frac{\partial k_1}{\partial x} + (q_1 r_1 + q_2 r_2)\frac{\partial k_1}{\partial y} \\ &\qquad + (p_2 r_1 - p_1 r_2)\frac{\partial k_2}{\partial x} + (q_2 r_1 - q_1 r_2)\frac{\partial k_2}{\partial y} = 0, \\ &(r_1^2 + r_2^2)\frac{\partial k_2}{\partial z} + (p_1 r_2 - p_2 r_1)\frac{\partial k_1}{\partial x} + (q_1 r_2 - q_2 r_1)\frac{\partial k_1}{\partial y} \\ &\qquad + (p_1 r_1 + p_2 r_2)\frac{\partial k_2}{\partial x} + (q_1 r_1 + q_2 r_2)\frac{\partial k_2}{\partial y} = 0. \end{aligned} \tag{3}$$

There exist unique solutions k_1, k_2 of the equations (3), analytic in the neighborhood of (x_0, y_0, z_0), taking for $z = z_0$ given values $m_1(x, y)$, $m_2(x, y)$, and possessing in (x_0, y_0, z_0) given partial derivatives $(\partial k_1/\partial x)_0$, $(\partial k_1/\partial y)_0$, $(\partial k_2/\partial x)_0$, $(\partial k_2/\partial y)_0$, provided that the functions to which $\partial k_1/\partial z$ and $\partial k_2/\partial z$ are actually equal in equations (3) are themselves analytic in the neighborhood of $(x_0, y_0, z_0, (\partial k_1/\partial x)_0, (\partial k_1/\partial y)_0, (\partial k_2/\partial x)_0, (\partial k_2/\partial y)_0)$,* that is to say, provided that the vectors c_1, c_2 are

* Cf., e. g., Goursat, Cours d'Analyse mathématique, vol. 2, p. 363.

analytic in (x_0, y_0, z_0) and r_1 and r_2 are not both zero there. These solutions of (3) are, then, analytic except perhaps in points of singularity of c_1, c_2 and in points for which $r_1 = r_2 = 0$. But they are identical with certain solutions of the equations (2), solved for $\partial k_1/\partial y$, $\partial k_2/\partial y$,—solutions which are analytic except perhaps in points of singularity of c_1, c_2 and in points for which $q_1 = q_2 = 0$. Evidently, then, these solutions k_1, k_2, and hence the vectors $\gamma_1 = k_1c_1 + k_2c_2$, $\gamma_2 = k_2c_1 - k_1c_2$, are analytic except perhaps in points of singularity of c_1, c_2 and in points in which both c_1 and c_2 are indeterminate, that is, have all three components zero. Thus we have the theorem:

If the gradients c_1, c_2 of the functions F_1, F_2 in $F = F_1 + iF_2$, where $F_1 = F_1|[L]|$, $F_2 = F_2|[L]|$ are functions of the first degree of the space curve L, are in general analytic, the gradients γ_1, γ_2 of Φ_1, Φ_2 in $\Phi = \Phi_1 + i\Phi_2$, an arbitrary complex function of L of the first degree isogenous to F, are analytic save perhaps in points of singularity of c_1 or c_2 and points in which both these vectors are indeterminate.

The theorem still holds when the vectors c_1, c_2 are proportional. In this case k_1 and k_2 are both solutions of the equation

$$p_i\frac{\partial k}{\partial x} + q_i\frac{\partial k}{\partial y} + r_i\frac{\partial k}{\partial z} = 0, \quad (i = 1, 2),$$

to which both of the equations (3) reduce in form, and γ_1 and γ_2 are both proportional to c_1 and c_2.

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AN ELEMENTARY DERIVATION OF THE PROBABILITY FUNCTION.

BY CAPTAIN ALBERT A. BENNETT, C.A.R.C.

WE shall derive by means of elementary considerations the equation of the probability curve from the sequence of binomial coefficients. If the asymptotic form of $x!$ be obtained, the problem is very simple but none the less merits attention. The asymptotic form of $n!$, viz., $\sqrt{2\pi n}(n/e)^n e^{\theta/(12n)}$, $0 < \theta < 1$, might of course be taken for granted, but so far as is known

to the writer, no purely algebraic attempt to prove Stirling's formula has been successful. The proof here given does use ratios of factorial expressions but proves the asymptotic values of these directly. The original derivation by Gauss is elegant and elementary but does not attempt to show the relation of probability in the finite cases to the transcendental probability function $ce^{-h^2x^2}$ nor does that derivation appear to adapt itself to any modification which will render obvious the fact that the infinite case is but a limit obtained from the finite case.

It is true that the method here described makes use of the convergence and the value approached of a single infinite product, viz., Wallis's formula for $\pi/2$,

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \cdots,$$

but this infinite product is a familiar one and is discussed in nearly every textbook treatment of infinite products.

Let a succession of adjacent rectangles R_k with altitudes, $\binom{2m}{m+k} a_m$, and common length of base, b_m, be erected on a straight line. The total area of these rectangles for a given m, with k ranging, is, by the binomial theorem, $2^{2m}a_m b_m$. If a_m and b_m be so varied with m that the middle rectangle R_0 remains of finite height and the area remains finite, then the rectangles form approximations to the probability curve. For the mid-ordinate $\binom{2m}{m} a_m$ to be finite, a_m must be asymptotically proportional to the reciprocal of the middle term $\binom{2m}{m}$. We seek therefore the asymptotic form of the middle term M_m. Let $C_m = mM_m^2/16^m$; then $C_{m+1}/C_m = (2m+1)^2/2m(2m+2)$. Now $C_1 = \frac{1}{4}$, hence, in general, for $m > 1$,

$$C_m = \frac{1}{2}\left[\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{4} \cdot \frac{5}{4} \cdot \frac{5}{6} \cdots \frac{(2m-1)^2}{(2m-2)2m}\right].$$

Hence $M_m \sim 4^m/\sqrt{m\pi}$, and apart from an arbitrary constant factor, $a_m \sim \sqrt{m\pi}/4^m$. Since the area $2^{2m}a_m b_m$ is also to be finite, we have, except for an arbitrary constant factor, $b_m \sim 1/\sqrt{m}$.

If a definite point x be chosen upon the base line, this point will fall within the rectangle numbered k only if kb_m does not differ from x by as much as b_m; the k denoting a fixed x must therefore vary with m, and we have $x \sim kb_m$, or $k \sim x\sqrt{m}$.

The asymptotic ratio of the general term $\binom{2m}{m+k}$ to the middle term $\binom{2m}{m}$ may be written in the form

$$\binom{2m}{m+k}\Big/\binom{2m}{m} = \frac{1}{1+\frac{k}{m}} \cdot \frac{1}{1+\frac{k}{m-1}} \cdots \frac{1}{1+\frac{k}{m-k+1}}$$

$$\therefore \left[\frac{1}{1+\frac{k}{m}}\right]^k > \binom{2m}{m+k}\Big/\binom{2m}{m} > \left[\frac{1}{1+\frac{k}{m-k+1}}\right]^k.$$

Replacing m partially in accordance with $k \sim x\sqrt{m}$, we have

$$\left(\frac{1}{1+x^2/k}\right)^k > \binom{2m}{m+k}\Big/\binom{2m}{m} > \left(\frac{1}{1+x^2/(k-x^2+x^2/k)}\right)^k.$$

Using the fact that the reciprocal of the middle term is asymptotically equal to the altitude factor a_m, we obtain by passing to the limit for m and hence also for k the asymptotic form of the height of the rectangle covering or touching the point x, viz., $y(x) = e^{-x^2}$.

We have thus obtained the equation $y = e^{-x^2}$ as the limit of the altitude of a rectangle contained in a sequence erected proportional to the binomial coefficients, and by different units of proportionality we would have obtained the form $y(x) = ce^{-h^2x^2}$, geometrically no more general. We have incidentally obtained the area $\int_{-\infty}^{+\infty} e^{-x^2}dx = \lim 4^m a_m b_m = \sqrt{\pi}$, directly, without recourse to such devices as polar integration of a double integral. Even the binomial theorem may be interpreted in the limit, giving

$$f(x\sqrt{2}) = \sqrt{\pi}/2 \int_{-\infty}^{+\infty} f(x+t)f(x-t)dt,$$

where $f(t)$ is e^{-x^2}.

This treatment is believed to be original, but the literature available for examination by the author is that customary to an army post, "somewhere on the Gulf of Mexico,"—nil.

THE INTRINSIC EQUATION FOR EULER'S RESISTANCE INTEGRAL.

BY PROFESSOR G. H. LIGHT.

EULER, in his Scientia Navalis, Proposition 53, gives a problem whose solution leads to the resistance integral

$$I = \int_{t_0}^{t_1} \frac{y'^3}{x'^2 + y'^2} dt. \tag{1}$$

This problem is as follows: Among all curves AM which with the axis AP and perpendicular PM comprehend the same area, to find that one which with its symmetric branch on the opposite side of the axis AP will form the figure offering the least resistance in water, when it moves in the direction PA along the axis.

This is a problem of the isoperimetric type where the integral I to be minimized is the integral (1) and the condition imposed is that the area between the axis and the curve is fixed, i. e.,

$$G = \int_{t_0}^{t_1} yx' dt = l.$$

The solution of this problem is known to be

$$x = \frac{1}{\lambda} \frac{p^2 - 1}{(p^2 + 1)^2} + \alpha, \quad y = \frac{1}{\lambda} \frac{2p^3}{(p^2 + 1)^2} + \beta,$$

where $p = y'/x'$ and α, λ and β are constants. Since this curve can always be obtained by a similarity transformation from the curve

$$x = \frac{p^2 - 1}{(p^2 + 1)^2}, \quad y = \frac{2p^3}{(p^2 + 1)^2}, \tag{2}$$

the general properties of the extremal can be obtained from the latter.

So far as is known, the exact nature of the extremals has never been determined. It is simply stated that they are rational curves of the fourth order with three cusps. The object of this paper is to show that when their intrinsic

equation is obtained, it is seen that the curve is a hypocycloid of three cusps. This is deduced in the following way:

The radius of curvature is found from (2) to be

$$R = \frac{2p(3-p^2)}{(1+p^2)^{3/2}}. \tag{3}$$

The length of arc is

$$S = \int_0^p \frac{2p(3-p^2)}{(1+p^2)^{5/2}}\,dp = -\frac{8}{3}\frac{1}{(1+p^2)^{3/2}} + \frac{2}{(1+p^2)^{1/2}} + \frac{2}{3},$$

from which

$$\frac{3}{2}\left(S-\frac{2}{3}\right) = \frac{3p^2-1}{(1+p^2)^{3/2}}. \tag{4}$$

In order to eliminate p from equations (3) and (4), form the expressions $R^2/4$ and $\frac{9}{4}(S-\frac{2}{3})^2$. It is seen at once that

$$\frac{R^2}{4} + \frac{9}{4}\left(S-\frac{2}{3}\right)^2 = 1, \quad \text{or} \quad R^2 + 9\left(S-\frac{2}{3}\right)^2 = 4, \tag{5}$$

which is the intrinsic equation of a hypocycloid. The curvature of (2) is

$$k = \frac{(1+p^2)^{3/2}}{2p(3-p^2)}, \tag{6}$$

also

$$\frac{dx}{dp} = \frac{2p(3-p^2)}{(1+p^2)^3}, \quad \frac{dy}{dp} = \frac{2p^2(3-p^2)}{(1+p^2)^3}. \tag{7}$$

From (6) and (7) it is seen that when $p = 0$ or $\pm\sqrt{3}$, dx/dp and dy/dp become zero and $k = \infty$. Hence (5) is the equation of a hypocycloid of three cusps.

UNIVERSITY OF COLORADO.

HERMITE'S WORKS.

Œuvres de Charles Hermite. Publiées sous les auspices de l'Académie des Sciences par EMILE PICARD. Vol. IV. Paris, Gauthier-Villars, 1917. 8vo. 593 pp.

THE present volume brings to a close the Œuvres of Hermite. It contains about ninety papers arranged chronologically, dating from 1879 and continuing to the year of his death, 1901.

By a strange coincidence, his first and last papers appeared in the initial volumes of new journals. The *Nouvelles Annales de Mathématiques* founded in 1842 contained the first fruits of his budding genius. Nearly sixty years later his last paper was written to lend the support of his great name to a new enterprise, *Le Matematiche pure ed applicate*. It was only a short note, a sort of open letter to the editor, which terminates with these words: "Prochainement je vous enverrai quelques remarques complémentaires, et une petite Note sur le produit d'une série quelconque par l'exponentielle e^{-x}." He did not live to carry out his benevolent intentions, but it is pleasant to recall that his last efforts were made in behalf of the rising generation and that he wished them to enjoy the encouragement that early publication and recognition alone affords.

The papers of this last volume are for the most part short and treat isolated topics in the elliptic functions, the Γ function, the functions of Legendre, continued fractions, and the theory of numbers. They represent the labors of an aging hero whose exalted official position makes exacting demands on his time and strength, but whose active brain still takes delight in research.

As one looks over the volume, there is much that catches the eye, but the fragmentary nature of most of the papers makes it impossible to give any adequate account of them without undue expenditure of time and space. Perhaps the most noteworthy are two papers on the application of the elliptic functions to the theory of numbers, viz., those beginning page 138, and page 223. They are a continuation of the line of thought which Hermite employed in his celebrated letter to Liouville, "Sur la théorie des fonctions elliptiques et ses applications à l'arithmétique" (1861). Instead of using the factors of the modular equation in the case of complex multiplication to get relations between the class numbers of binary quadratic forms of negative determinant as Kronecker had done, Hermite showed how the comparison of different developments of Θ quotients led to similar results. This method also gave him the number of decompositions of an integer into a sum of three and five squares, a problem rendered celebrated by the researches of Henry Smith and Minkowski.

A paper of general interest is entitled "Sur les racines de la fonction sphérique de seconde espèce," i. e., the roots of

$Q_n(x) = 0$. It was written in 1890, the same year as Klein's paper on the roots of the hypergeometric function $F(\alpha, \beta, \gamma, x)$. Neither author knew of the other's work at the time of publication, moreover the methods employed are totally different. Hermite's work on $Q_n(x) = 0$ led Stieltjes* to attack the same problem from another and more direct point of view. Neither paper seems to have received the attention it deserves.

The many and interesting properties of the Γ function came in for a large share of Hermite's attention. We note as especially important two papers on this subject: the one entitled "Sur une extension de la formule de Stirling," page 378, composed in 1893, the other "Sur la fonction log $\Gamma(a)$," page 412, written in 1895, and dealing with the same problem, viz., the asymptotic expansion of log Γ.

But of all the subjects dear to Hermite in his later years, the elliptic functions without doubt held first place. He knew Jacobi's Fundamenta Nova by heart and he reveled in the endless maze of formulas which to others seem almost an intolerable burden. Here are a few titles dealing with these functions: "Sur la différentiation des fonctions elliptiques par rapport au module," "Sur l'intégration de l'équation différentielle de Lamé," "Sur une proposition de la théorie des fonctions elliptiques," "Sur une formule d'Euler," "Sur une représentation analytique des fonctions au moyen des transcendantes elliptiques." These will suffice to give the reader an idea of the rest.

A feature of this volume will appeal to all mathematicians, as it reveals in no uncertain way Hermite's generous and affectionate nature; we mean the short sketches of the careers of his departed friends and colleagues, Bouquet, Halphen, Kronecker, Kummer, Cayley, Weierstrass and Brioschi. Speaking of Kronecker he says: "La louage se tait devant le deuil de la Science et l'émotion causée dans tout le monde mathématique, par la perte cruelle du grand Géomètre; à ces regrets douloureux, à ces souvenirs d'une vie remplie par tant de travaux et de découvertes, je joins ceux d'une amitié qui a été pendant trente années, l'honneur de ma vie scientifique et que je ne retrouverai plus."

Of Weierstrass he writes: "La vie de notre illustre Confrère a été en entier consacrée à la Science qu'il a servie avec un absolu dévoument. Elle a été longue et comblée d'honneurs;

* *Annales de Toulouse*, vol. 4 (1890).

mais devant une tombe qui vient de se fermer, nous ne rappelons que son génie et cette universelle sympathie qui s'accorde à la noblesse du caractère. Weierstrass a été droit et bon; qu'il reçoive le suprême hommage plein de regrets et de respect que nous addressons à sa mémoire! Elle vivra aussi longtemps que des esprits avides de vérités consacreront leur efforts aux recherches de l'Analyse, au progrès de la science du Calcul."

Let us quote from one more éloge, that of Brioschi: "J'ai été associé aux travaux de Brioschi; nous avons souvent mis en commun nos efforts; j'ai suivi sa carrière qui a été si belle, remplie par l'étude, et de grands services rendus à son pays. Nul ne ressent plus que moi la perte du grand géomètre, et de l'homme d'honneur, le souvenir de son amitié, d'une étroite liaison remontant à notre jeunesse me restera à jamais l'un des meilleurs et des plus chers de toute ma vie."

Other interesting biographical matter is contained in an address by Hermite at the inauguration of the new Sorbonne in 1900. It gives a brief but illuminating analysis of the work of the chief French mathematicians in the first half of the nineteenth century.

Finally we note the touching discours pronounced by Hermite at the celebration of his seventieth anniversary in 1892. A photograph of the medallion by Chaplain presented to him on this occasion by his friends and admirers from all over the world forms the frontispiece of the volume and a facsimile reproduction of an extract of a letter to Tannery relative to Hermite's classic researches on the elliptic modular function in 1858 brings the volume to a fitting close.

JAMES PIERPONT.

BLICHFELDT'S FINITE COLLINEATION GROUPS.

THE author of "Finite Collineation Groups" is certainly under great obligations to Professor H. H. Mitchell for his very generous and thorough review in this BULLETIN of February, 1918; particularly so since Professor Mitchell had evidently read the book with great care. The author is in full accord with the reviewer on a number of the defects pointed out by the latter; and he offers herewith a few remarks in the hope of clearing up those statements in the book that

seemed inexact or obscure to him upon reading Professor Mitchell's review. Unless otherwise stated, page and paragraph references are to the book in question.

1. (BULLETIN, page 247.) Kronecker's theorem was originally stated correctly by the author (*Transactions of the American Mathematical Society*, 1903, pages 390–391; *Mathematische Annalen*, 1907, page 558 (in this place, line 3 from the bottom, the inequality $t < p$ should read $t < p^{n-1}$)). The incorrect form occurs in *Mathematische Annalen*, ibid., page 559; in Finite Groups by Miller-Blichfeldt-Dickson, page 240; and in the book under discussion, page 187. After making the proper corrections in the sentence beginning "Kronecker's theorem implies" (page 187, lines 3–5), namely by substituting for "k roots" the following: "the roots of this equation (either directly or after adding a certain number of pairs of mutually cancelling roots of unity)," the author would leave the next sentence unaltered, and then introduce a new sentence (line 8) to read: "In no case need p be taken greater than k." The remaining portion of 6° will stand as it is. Under 7°, the sentence beginning (line 6 from the bottom) "More definitely" should be replaced by "In particular, if the total number of roots in N equals k, then all these roots are equal, or $N = 0$; if the number of roots is $< k$, then $N = 0$."

2. (BULLETIN, page 249.) In the chapter on group characteristics the meaning of the term "equivalent" is in accordance with the usage in the writings of Burnside, Frobenius, and Schur, though these authors say "equivalent representations of a group" instead of "equivalent groups." Two simply isomorphic groups having their operators so arranged that S_i in the one corresponds to T_i in the other, $i = 1, 2, \cdots, g$, are equivalent if a change of variables (transformation) V exists such that $V^{-1}S_iV = T_i$ for every subscript i. The same two groups may possibly not be equivalent when a new one-to-one correspondence is set up in a different manner. The author has, admittedly, used the word freely in the chapters on the binary, ternary and quaternary groups without calling attention to the fact that in these chapters, if the phrase "non-equivalent" were used in its strict sense, we should have, for example, two non-equivalent primitive binary groups (or two non-equivalent representations of the group) of order 60φ; namely the group (E), page 73, and that group whose trans-

formations are obtained from the corresponding transformations of (E) by interchanging $+\sqrt{5}$ and $-\sqrt{5}$ (this cannot be brought about by a change of variables).

3. (BULLETIN, page 250.) That a primitive group in four variables cannot contain a transformation of order 9 follows from § 117, the reference to this fact being omitted by an oversight. By the reasoning of § 111 it is found that a tentative transformation of order 9 generates a Sylow subgroup of this order, and the presence in a primitive simple group of such a subgroup would exclude all the types given in § 111 except (a) and (g). Since groups of order $2^a \cdot 3^b$ are not simple, the primitive group in question could only be of the kind discussed in § 117.

4. (BULLETIN, page 250.) Concerning the statement "In view of . . . appears superfluous," the author remarks that as yet (§§ 106–108) it had not been shown that T, C, D, V, F generate a *finite* group; i. e., K', containing K as a subgroup, might as yet prove not to be of finite order. Hence the necessity for showing that K' and K are identical, and this is in part the purpose of § 109 (cf. §§ 115, 116, where the same problem is encountered for other groups).

5. The author hereby takes opportunity to emphasize that throughout the chapter on group characteristics, the term "simple group" has its customary meaning; i. e., a group which contains similarity transformations is not "simple" here, though it be given that the corresponding collineation group is simple. (Cf. § 51, 1°, concerning the use of the term in the chapters on groups in 2–4 variables.)

6. In conclusion the author submits the following additional list of errata (the first two were suggested by Professor F. N. Cole):

Page 18, line 1, and page 55, line 16. Read "any set" instead of "any one set."

" 129, " 16. Corollary 2 should read "Let . . . characteristics of two simply isomorphic groups . . ." instead of "Let . . . characteristics of two groups. . . ."

" 70, " 14. The equation $\alpha^\varphi = 1$ should be $\alpha^{3\varphi} = 1$.

" 73, " 6 above § 59. After $T'^2 = E_1$, insert: $S'T' =$ a transformation of order 3φ.

" 104, " 5 from bottom. Replace the first x_2' by x_1'.

" 121, last line. Replace $(x_1) = \alpha x_1$ by $(x_1)M = \alpha x_1$.

" 126, line 7 from bottom. Replace f by cf.

" 127, " 8. The letters $\overline{X}_1, \ldots, \overline{X}_n$ should be accented: $\overline{X}_1', \ldots, \overline{X}_n'$.

Page 133, in Exercise 1, after ". . . consists of" insert "the group represented by E repeated 60 times."
" 138, footnote. Replace 6° by 7°.
" 182, line 20. The second of the set of numbers given, namely 6, should be 3.
" 182. At end of footnote marked * write: Jordan's method is to express suitable powers of the solutions of the linear differential equation as rational functions of x and of all the roots of a certain algebraic equation. The degree of this equation will then generally be lower than the corresponding degree found above. See *Crelle*, vol. 84, pp. 93, 112.

H. F. BLICHFELDT.

REMARKS ON ELLIPTIC INTEGRALS.

IT is known that an elliptic integral of the first kind is everywhere one-valued, finite, and continuous on its associated Riemann surface, while the elliptic integral of the second kind is algebraically infinite, and the elliptic integral of the third kind is logarithmically infinite at certain points of the surface. This is a characteristic distinction of these integrals and is essential in their study. It is also true of the hyperelliptic and abelian integrals.

The Legendre form of the integral of the first kind is

$$F(k, \varphi) = \int_0^{\phi} \frac{d\varphi}{\sqrt{1 - k^2 \sin^2 \varphi}}.$$

When $k = 1$, this integral becomes

$$F(1, \varphi) = \int_0^{\phi} \frac{d\varphi}{\cos \varphi} = \log \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right).$$

If further $\varphi = \frac{1}{2}\pi$, it is seen that the complete elliptic integral

$$F_1(1) = F\left(1, \frac{\pi}{2}\right)$$

is logarithmically infinite, while $F_1(0) = \pi/2$.

As this is the only possible chance, remote though it be, for an integral of the first kind "to claim kin" with one of the third kind, I don't see why a gentleman from Alabama, where relationships are cherished, the connection often being even more remote, should suffer a "jolt" (see the BULLETIN, Febru-

ary, 1918, page 253) when I write "F_1 increases from $\pi/2$ to logarithmic infinity."

I am glad, however, that the "logarithmic" injection left him so dazed that he did not notice the glaring error found a few pages farther on in the monograph on Elliptic Integrals. I take this opportunity of correcting it. On page 33, line 12 below, is found "in the formulas $\operatorname{sn} iu = i \operatorname{tn}(u, k')$, etc., write $u + iK$ for u." This obviously should be "write $u + K'$ for u" with the resulting fundamental formula

$$\operatorname{cn}(iu + iK', k) = -\frac{1}{k}\frac{\operatorname{dn}(u, k')}{\operatorname{sn}(u, k')}.$$

This is found correctly given in my larger book, page 464, and follows at once from formulas (XIX) and (XVII) of pages 248 and 247 of that work. The formula is also correctly derived in two different ways in my lecture notes, from which I thought the monograph was taken. This leads me to suggest that an author be allowed one bad mistake for every 100 pages, the same to be classified under the heading "inexplicables, lapses, aberrations, etc."

May I also add that in my lectures the pronoun "we" without intentionally implying anything personal is perhaps too frequently used? At any rate one of the editors of the monograph in question thought this to be the case. To oblige him the other editor and I suppressed some of the "we's" and it appears that we did not make other corresponding changes in at least two places. Thus two infelicitous "grammatical connections" remain. I am obliged to Professor Carmichael for not characterizing them more harshly.

HARRIS HANCOCK.

THE UNIVERSITY OF CINCINNATI.

SHORTER NOTICES.

Differential Calculus. By H. B. PHILLIPS. New York, Wiley, 1916. 162 pp.

Integral Calculus. By H. B. PHILLIPS. New York, Wiley, 1917. 194 pp.

To expound a few central methods and apply them to a large variety of examples to the end that the student may learn

principles and gain power is the plan which the writer announces in the preface to this text. A perusal of the book leads to the conclusion that there is justification for this claim. The explanations are brief and simple and the problems offer variety and appear to be well graded. The question of answers is met by furnishing them at the end of each book, for the problems in the chapters, and adding to each a set of over two hundred exercises without answers. Although the differential and integral calculus form two separate books (which are bound separately or together), the arrangement of chapters is such that it would be easy for a teacher who desired to do so to bring in the simpler work in integration before the last chapters on differentiation. The order of subjects in the differential calculus is to be noted. Chapter II is on the derivative and the differential. Some may object that the introduction of the latter before the student has had time to assimilate the idea of the former will cause confusion. There is, however, room for difference of opinion in this matter. After the differentiation of algebraic functions and a brief discussion of rates, there is a chapter on maxima and minima, with simple concrete problems. It is not until after the chapter on differentiation of transcendental functions that there is a consideration of tangents and normals, angles between curves, points of inflection, curvature, etc. The differential calculus is completed by chapters on velocity, series, and partial differentiation. In the book on integral calculus in Chapter II (on formulas and methods of integration) there is a noteworthy omission of those "ingenious devices" that are so dear to some writers of texts and that are so apt to inspire a beginner with the feeling that success in integration is a matter of luck rather than of knowledge of principles. After a short and simple explanation of definite integrals, there are chapters on simple areas and volumes, other geometrical applications, mechanical and physical applications, approximate methods, double and triple integrations, and, finally, a longer chapter on differential equations. The book is concluded by a brief table of integrals and a table of natural logarithms.

E. B. Cowley.

Modern Business Arithmetic, Complete Course. By H. A. FINNEY and J. C. BROWN. New York, Holt, 1917.

THE book under review is a modern up-to-date text on the arithmetic of modern business, intended primarily for use as a text in commercial schools and business colleges.

The book has a number of features of interest to the mathematician as well as to the teacher of elementary mathematics. On the purely arithmetical side the most important feature is the emphasis placed on drill in correct methods of computation as essential to the acquirement of facility and accuracy, which means efficiency, in practical work. Numerous exercises, both oral and written, to be completed within a certain time limit, are given for the purpose of developing speed. Various checks, such as casting out the nines, and also special short methods are given, and the pupil is taught to check his results at every step. This aspect of the work is especially important in grade work, as most children rely on an answer book or verification by the teacher for the proof of their work, and have no confidence in their own work or means of determining its correctness. This lack of initiative and self-confidence constitutes a serious defect in our present training in arithmetic which the methods adopted in this book would assist materially in correcting.

On its applied side, the work, although comprised in less than 500 pages, is a veritable compendium of modern business practice, and will prove a valuable reference book for teachers of seventh and eighth grade arithmetic. Ruled forms and tabulated business statistics are freely used, in order to put the pupil in touch with the actual demands of the business world.

The first fourteen chapters, covering 152 pages, are devoted to the fundamental processes of arithmetic, including percentage, and the use of graphs for presenting business statistics. Numerous exercises are included, many of which are purely arithmetical, while others are stated in the form of problems met in business practice.

The remainder of the book is devoted to technical commercial applications. Chapters 15–23, covering 79 pages, relate to trading activities, including such topics as profit and loss, recording sales, paying for goods, collecting bills and taking inventory.

Chapters 24–28, comprising 34 pages, are on borrowing and loaning, and include the modern field of contract purchases and installment payments.

Chapters 29–36, covering 54 pages, are devoted to business expenses, such as wages and payrolls, insurance and taxation, and include such modern topics as depreciation, advertising and the income tax.

Chapters 37–60, covering 19 pages, are concerned with business organization, partnership, corporations, insolvency and bankruptcy.

Chapters 41–45, occupying 40 pages, relate to business efficiency, and form the most up-to-date section of the entire book. The nature of this section is apparent from the list of topics covered, which include factory costs, buying and selling expenses, determination of profit and loss by departments and for separate sales, and tabulations for the sales manager such as department and salesmen's records.

Chapters 46–48, occupying 12 pages, conclude the work with miscellaneous topics not previously treated, such as consignments and commissions, life insurance and farm management.

The impression made by this book is that the authors have succeeded in producing not only an unusually complete and well written treatise on business arithmetic, but also a work that is eminently teachable, as it is bright and interesting, and well arranged for classroom purposes.

S. E. SLOCUM.

Plane Trigonometry with Tables. By E. H. BARKER. Philadelphia, P. Blakiston's Son and Co., 1917. 172 pp.

BARKER'S Trigonometry treats, in its few pages, the usual topics of the subject. The functions of an acute angle are defined as ratios in the first chapter, the functions of a general angle being postponed to Chapter IV, while the intervening chapters deal with the relations between the functions of an acute angle. For the graphs the line values of the functions are used. The addition formulas are proved for acute angles, without the use of directed lines. Geometrical proof is given for the laws of sines and cosines, and analytical proof for the law of tangents. Some unusual formulas are given in the work on solutions of triangles. The ninth chapter, on circular measurement of angles and inverse identities, completes

the work on trigonometry. The definition, explanation and use of logarithms are given in Chapter X.

Since there is such a variety of trigonometry texts on the market, one looks critically at each new text to see what good points warrant adding one more book to the already large supply. Among the virtues possessed by Barker's Trigonometry we note its small amount of detail in general, reinforced, however, by a wealth of applications in some cases where familiarity with the ideas can be gained only by repeated practice with them. Occasionally the author seems to lose sight of this apparent ideal of brevity, and gives details certainly unnecessary for a college freshman and probably equally unnecessary for any student of trigonometry. Yet, in other places the student is given credit for power which few students of trigonometry would possess.

The author gives some excellent advice to students, especially in urging the fullest use of given material in preference to computed material, in suggesting a reasonably accurate figure as a guide to the solution of a triangle and as a check on results, and in emphasizing checks for results in general. Throughout, the text offers a large number of examples chosen from varied material, and often useful for developing the initiative of the student.

The author has given "special attention to the preparation of the tables and has designed the tabular arrangement to give the computer maximum efficiency with a minimum of labor." In two respects the opposite of this aim seems to be the result of this special attention. First, the entire five digit mantissa is repeated for each number, giving the pages of the logarithms of numbers a most compact, dense appearance which is hard on the eyes of the computer. Second, the natural functions are given in two separate tables, one for sines and cosines, one for tangents and cotangents, thus often requiring double work in securing two functions of the same angle.

The introductory explanation of the theory of logarithms is clearly given with a well chosen minimum of material. However, it is surprising to find advice to the student to approximate with a four-digit number by taking the preceding four-digit number instead of the nearest four-digit number. The use of four-digit approximation is so frequent throughout the book that the student has very little practice in interpolation.

In fact, the book does not at all help the student to realize the ideal expressed by the author, that "the student should at all times strive to get answers as nearly accurate as the tables will permit." Nothing whatever is said about the degree of accuracy warranted by the given statistics of a problem.

Errata are numerous in the text. Some parts of the book are poorly written. There is a dearth of punctuation in general, and of periods in particular. In places the arrangement of the material is very poor, while in other portions of the text the material is well spaced and the page presents its salient points at a glance. The figures and curves are very good.

M. E. WELLS.

Tables numériques usuelles. Par L. ZORETTI. Paris, Gauthier-Villars, 1917. 8vo. 52 pp. 3 fr.

THESE small tables are designed for the use of engineers, students of college and lower grades, and general computers. They consist of multiplication tables for the multipliers 1, 2, $\cdots$, 9 and multiplicands of three figures; together with columns that give for three figure arguments the reciprocal, the square and the common logarithm to four places. A table of trigonometric functions, grades, and radians for every degree and quarter degree follows. Tables of logarithms of certain common constants, and the values of πd and $\frac{1}{4}\pi d^2$ for two figure arguments close the volume. The leaves are thumb-indexed and by proper spacing made otherwise convenient. The type is clear and the paper with no glare. The book is ample for most of the computations involved in the author's general course in mathematics.

JAMES BYRNIE SHAW.

Ueber gewisse Teilbereiche und Erweiterungen von Ringen. By Dr. Phil. ADOLF FRAENKEL. Leipzig und Berlin, Teubner, 1916. 64 pp.

IN the investigations of the arithmetic properties of the algebraic numbers the fundamental notion is that of domain of rationality, in which the four operations of arithmetic, excluding only division by zero, are permissible. Besides these certain other sets, called orders by Dedekind and rings

by Hilbert, have been used. In these rings the operations addition, subtraction, and multiplication are permissible, but division is not always possible and, when possible, does not always lead to a unique result.

These rings are special types of groups, where the term group is used in the more general sense of the French edition of the mathematical encyclopedia. The first general investigation into their nature was made by the author of the present work in his dissertation, published in *Crelle's Journal*, volume 145. In this investigation he makes frequent use of the group property and in the proofs of some theorems he shows that the elements under consideration form a group in the ordinary sense and draws his conclusions from known theorems in the theory of groups.

The present work is a further study of certain types of rings, with special reference to certain subsets, and extensions formed by the adjunction of new elements.

Two elements a and b of a ring R are called equivalent if a is divisible by b and b by a. A divisor of zero is an element b such that $a \cdot b = 0$ when $a \neq 0$. An element which is not a divisor of zero is a regular element. A divisor of zero which, aside from regular elements, has no divisor except itself is called a prime divisor. A simple ring is one which contains only one prime divisor.

The work is devoted to the consideration of rings in which multiplication is commutative and in which the following two conditions are satisfied:

I. There are only a finite number of non-equivalent divisors of zero.

II. If $a \cdot b$ is divisible by c, then there exist two elements c_1 and c_2 such that $c = c_1 \cdot c_2$ and a is divisible by c_1 and b by c_2.

Rings which satisfy these conditions are called decomposable, because from the elements of such a ring R, in which there are n non-equivalent prime divisors, we may construct n simple rings $R_1, R_2, \ldots, R_n$, such that no element of R, except zero, is contained in more than one of the rings R_i, the product of two elements from two different rings R_i and R_j is zero, and every element of R has a unique representation as the sum of n elements, one from each of the rings R_i.

This property is of fundamental importance in the theory of the g-adic numbers which constitute a ring such as has been defined above.

A partial ring $\overline{R}$ of a ring R is a set of elements of R, closed with respect to addition, subtraction and multiplication, but in which the equation $ax = b$ has a solution only when it has a solution in R. Since the author has chosen to call a ring a set of elements in which division by regular elements is always possible and a may be a divisor of zero in R but regular in $\overline{R}$, we see that $\overline{R}$ is not necessarily a ring.

A prime ring is a partial ring which does not contain as a subset another partial ring having the same unit element.

The special prime ring is that prime ring whose unit element is the unit element ϵ of R.

A part of the first chapter is devoted to the determination of the structure of the special prime rings. They are found to be of four types according as the sequence $\epsilon, 2\epsilon, 3\epsilon, \ldots$ contains: (1) neither zero nor divisors of zero; (2) zero but no other divisor of zero; (3) both zero and divisors of zero; (4) divisors of zero but not zero.

The concluding section of the chapter deals with prime rings in general and contains a theorem regarding the number of prime rings in a given ring and their structure.

To aid the reader in understanding the theory the author gives, at various points in the chapter, examples of rings chosen from systems of residues with respect to a certain modulus or from systems of complex numbers.

Chapter II consists of a study of the rational functions of x in a simple ring R. The degree of a polynomial is defined as usual. The order of a polynomial is the exponent of the highest power of x whose coefficient is a regular element of R. The degree of the product of two polynomials may be less than the sum of their degrees, but the order of the product is the sum of the orders.

A polynomial which has at least one regular coefficient is called a primitive polynomial. Every primitive polynomial of order n can be multiplied by a primitive polynomial of order zero so that the product will be of degree n. By applying this theorem the author discusses Euclid's algorithm in a simple ring. This process always leads to the highest common divisor if the successive remainders are all primitive, but fails if the process finally gives a remainder which is not primitive.

Chapter III contains a discussion of the extensions of a simple ring formed by the adjunction of new elements. An

extension formed by adjoining an element which satisfies an algebraic equation in R is called an algebraic extension. If the element which is adjoined does not satisfy an algebraic equation in R the extension is called transcendental.

From the simple transcendental extensions formed by the adjunction of the element x to R the author passes to the algebraic extensions by assigning to x a value which is a root of an algebraic equation.

Finally by making use of the theorem regarding the decomposition of rings the results found are extended to rings having more than one prime divisor.

G. E. WAHLIN.

Calcul des Systèmes Elastiques de la Construction. Par ERNEST FLAMARD. Paris, Gauthier-Villars, 1918.

THIS work is a treatise of 200 pages, published under the imprint of the "Encyclopédie Industrielle," on the application of the principle of least work to the calculation of the reactions and deflections of straight and curved beams, the elastic arch, and pin-connected structures having redundant members or supports.

The feature of most interest to the writer of this review is the evidence the work furnishes that there are mathematicians and engineers in France today of such mental poise that they are able to concentrate their attention on a purely theoretical question of method which brings out no new results and has absolutely no relation to the war or the future.

In this work it is first shown that the elastic forces acting on any section of a solid are reducible to three static elements consisting of a bending moment and normal and shearing forces. The expressions for the work of deformation due to these three elements are then derived, as well as Castigliano's well known theorem, giving the linear and angular displacements of the external forces and couples in terms of the partial derivatives of the work of deformation with respect to these elements. This is followed by the derivation of the principle of least work of deformation. These results are then extended to include the effect of change in temperature. In applying the results to beams under vertical loading, however, it is pointed out that the temperature forces are the only external forces acting parallel to the axis of the beam, and

may therefore be considered separately and the equation of restraint due to temperature effects written once for all.

The applications begin with the simplest cases of restrained beams carrying a uniform load or a single concentrated load, and by calculation of the work of deformation, the usual results for moments and reactions at the supports, and the linear and angular deflections, are obtained.

Next in order come continuous beams of n spans with fixed ends and carrying either a uniform load in each span or a single concentrated load in one span. The principle of least work is first used to derive the theorem of three moments, which in the most general case takes the form of a quadrature. The ordinary form of this theorem for constant cross section is then deduced from the general expression as a special case, and the expressions for the linear and angular deflections in any segment are obtained in terms of the derivatives of the work of deformation.

The treatment of continuous beams is followed by the application of the same general method to curved beams, that is, beams of given curvature and having a continuous tangent, loaded uniformly over the chord, or horizontal projection, of the curve. The horizontal and vertical reactions and the moments at the supports are first obtained by applying the method of least work. The linear deflection at any point is then determined by applying a given load at this point, calculating the corresponding deflection from the partial derivatives of the work of deformation with respect to this load, and reducing the load to zero in the resulting expression. The angular deflection at any point of the beam is also obtained in the same way by applying a given couple and after using it for evaluating the derivatives of the work of deformation, reducing it to zero in the resulting formula.

Special cases are obtained from the general formulas for a beam bent into a circular arc with constant cross section, and for different conditions of end restraint. The entire process is then repeated for a curved beam carrying a single concentrated load.

The last section is an application of the general methods to statically indeterminate pin-connected systems or jointed linkworks. The treatment is divided into four cases: (1) when the system contains redundant supports; (2) when it contains redundant members; (3) when it includes redundant

supports as well as redundant members; (4) mixed systems, such, for example, as a trussed continuous beam.

The method of attack consists in first separating the statically indeterminate system considered into two separate systems such that each is statically determinate. The principle of least work is then applied by equating the derivatives of the total work of deformation with respect to the reactions common to the two systems to zero, and from these conditions determining the common restraints.

To find the deflections, a unit load is placed at any specified point and the deflection is obtained from the partial derivatives of the work of deformation with respect to this unit load. To shorten the process, the work of deformation is expressed in terms of influence numbers which represent the stress in any member due to the various forces acting. These influence numbers are subsequently determined graphically by drawing a separate stress diagram for each of the applied forces or restraints.

The work throughout is mathematically rigorous, and marks a beginning in an important field, as the method of least work, although familiar as a general principle, has not been used to any extent in the theory of elasticity, and promises a complete solution of many problems which have so far received inadequate treatment. S. E. SLOCUM.

Annuaire pour l'An 1918. Publié par le Bureau des Longitudes. Paris, Gauthier-Villars, 1918.

AMONG the "Notices" of the *Annuaire* for 1918 is a timely one by M. J. Renaud, entitled "L'heure en mer." A general revision of our standards of time is one of the by-products of the war. Europe and America have largely adopted the plan of moving the clock forward one hour in summer, thus recognizing the fact that most of our daily acts are much more closely associated with the numerical names of the hours than with the altitude of the sun. Astronomers still begin their day at noon rather than midnight, but here again there is a movement on foot to synchronize the commencements of the civil and astronomical days. The time at sea, where a vessel is continually changing its longitude and therefore its local time, requires different treatment; the older methods also of fixing the "ship's time" require some alteration in view of the advent of wireless telegraphy, which enables the navigator to

keep in constant touch with the land. "Local time" requires a knowledge of the ship's position and any record of events at sea is dependent at present for the hour on the accuracy of the local time. Various suggestions are considered by M. Renaud to avoid the difficulty. Perhaps the most practical is that ships' logs be kept in both local and Greenwich time and that all reports be furnished with the latter time only. Zone time is also perfectly practicable. The reader who has travelled much at sea can enjoy a relaxing hour in considering his personal preferences concerning the "how" and "when" of the daily alteration of the clock.

M. Bigourdan gives a full account of the Egyptian calendar; M. Maurice Hamy summarizes our present knowledge of the connection between solar phenomena and terrestrial magnetism and M. Emile Picard has a brief account of the life and work of Darboux. The last of these "Notices" will be read with interest as showing the varied activities of a scholar who is known to most of us solely as a pure mathematician.

Minor changes and improvements in the body of the volume are somewhat more numerous than usual. One useful feature is the addition in the Index of references to articles appearing in the three previous issues. It will be remembered that the growth of the Annuaire has required a division of the subjects, which are now treated in alternate years or at longer intervals. The new Index will save much trouble in searching previous volumes. The principal astronomical events and tidal data for 1919 are to be found in a supplement.

E. W. BROWN.

NOTES.

THE April number (volume 19, number 2) of the *Transactions of the American Mathematical Society* contains the following papers: "Proof that certain ideals in a cyclotomic realm are principal ideals," by H. H. MITCHELL; "The order of primitive groups (III)," by W. A. MANNING; "On the degree of convergence of Birkhoff's series," by W. E. MILNE; "Problems in the theory of ordinary linear differential equations with auxiliary conditions at more than two points," by C. E. WILDER; "Transformations of applicable conjugate

nets of curves on surfaces," by L. P. EISENHART; "A new integral test for the convergence and divergence of infinite series," by R. W. BRINK; "On the reduction of certain differential equations of the second order," by W. D. MACMILLAN; "Invariants of differential configurations in the plane," by E. F. SIMONDS.

THE concluding (June) number of volume 19 of the *Annals of Mathematics* contains: "A class of developments in orthogonal functions," by TOMLINSON FORT; "A formula of polynomial interpolation," by W. G. SIMON; "Plane nets with equal invariants," by G. M. GREEN; "Recent extensions of Descartes' rule of signs," by D. R. CURTISS; "A general form of integral," by P. J. DANIELL; "Elastic stresses in an infinite solid with a spherical cavity," by T. H. GRONWALL.

THE second annual Register of Officers and Members of the Mathematical Association of America has recently appeared as supplement to the May number of the *American Mathematical Monthly*. The Association has now a membership of 1,056, not including institutional members.

AT the meeting of the Edinburgh Mathematical Society held May 10, 1918, the following papers were read: By D. M. Y. SOMMERVILLE: (*a*) "Some applications of spherical geometry to geometrical optics", (*b*) "Note on the minimum duration of twilight"; by T. H. MILLER: "Note on the radical axis of intersecting circles."

IN the announcement of courses to be given at the University of Illinois (June BULLETIN, page 460), Professor G. A. MILLER's course on "Continuous groups" should have been scheduled for the first instead of the second semester. In the second semester Professor Miller will give a course on "Finite groups," which was not included in the announcement.

FEW personal sacrifices of this war can have been greater than that of Professor CAMILLE JORDAN, who has lost three sons and a grandson, killed by the enemy. The sympathies of all American mathematicians will go out to the man to whose teachings they owe so much and whom they all esteem.

THE following university and college teachers of mathematics have recently entered the national military service:

MR. RALPH KEFFER, of Harvard University, has enlisted in the aviation corps. Mr. F. H. MURRAY, of Harvard University, and Dr. G. A. PFEIFFER, of Princeton University, have joined the national army. Mr. C. H. PARSONS, of Columbia University, has joined the ordnance. Assistant Professor WARREN WEAVER, of Throop College of Technology, is serving in the science and research division of the signal corps at Washington.

PROFESSOR H. E. HAWKES, of the department of mathematics of Columbia University, has been made dean of Columbia College, Dean F. P. Keppel having resigned to become third assistant secretary of war. Professor Hawkes has been acting dean during the past year.

ASSOCIATE professor D. D. LEIB, of Connecticut College, has been promoted to a full professorship of mathematics.

MR. E. S. LANE, of the University of Chicago, has been appointed instructor in mathematics at Rice Institute.

MR. ALFRED DAVIS, of the Francis W. Parker School at Chicago, has been made head of the department of mathematics at William and Mary College.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

BEEGER (N. G. W. H.). Over eene functie, vooregesteld door eene reeks van Dirichlet. Proefschrift. Groningen, Gebr. Hoitsema, 1916. 8vo. 84 pp.

BORTKIEWICZ (L. VON). Die Iterationen. Ein Beitrag zur Wahrscheinlichkeitstheorie. Berlin, Springer, 1917. 8vo. 12 + 206 pp. Geb. M. 10.00

HUYGENS (C.). Œuvres complètes de Christian Huygens publiées par la Société Hollandaise des Sciences. 13 tomes in 14. La Haye, Nijhoff, 1888–1916. 4to. Couv. vélin. Fl. 180.00

D'OCAGNE (M.). Cours de géométrie pure et appliquée de l'Ecole polytechnique. Tome 2: Cinématique appliquée. Stéréotomie. Statique graphique. Calcul grafo-mécanique. Nomographie. Paris, Gauthier-Villars, 1918. 8vo. 6 + 364 pp. Fr. 18.00

PRINGSHEIM (A.). Vorlesungen über Zahlen- und Funktionenlehre. 1ter Band, 2te Abteilung: Unendliche Reihen mit reellen Gliedern. Leipzig, Teubner, 1916. 8vo. 8 + 222 pp. M. 10.80

SIERPINSKI (W.). Analiza. Tom 1. Cześć pierwsza: Liczby rzeczywiste i respolone. 14 + 237 pp. Cześć druga: Działania nieskończone. Moskwa, W. Lemana, 1916–1917. 8vo. 4 + 5 rubles

II. ELEMENTARY MATHEMATICS.

CHANKIN (J.). See STRASSBURGER (A.).

COMBEROUSSE (C. DE). Trigonométrie rectiligne et sphérique. Tables des valeurs naturelles des fonctions circulaires de minute en minute d'arc avec 6 décimales. 6e édition, entièrement refondue par R. de Montessus de Ballore. Paris, Gauthier-Villars, 1917. 8vo. Fr. 6.00

FAZZARI (G.). I numeri reali e l'equazione esponenziale $a^x = b$ per le scuole medie superiori. Palermo, Libreria Scientifica D. Capozzi, 1918. 4 + 75 pp. L. 1.80

FRIIS-PETERSEN (F.) og JESSEN (J. L. W.). Mellemskolens ny Regnebog. I–IV Hefte. Kφbenhaven, Gjellerup, 1917.

HELMICK (P. S.). Table of $\log_{10} \sec^2 \theta$. (University of Iowa Monograph, No. 4.) Iowa City, University of Iowa, 1917. 16 pp.

JAVELOT (G.). Comment résoudre les problèmes de géométrie élémentaire. Paris, F. Nathan, 1918. 12mo. Fr. 3.00

JESSEN (J. L. W.). See FRIIS-PETERSEN (F.).

MONTESSUS DE BALLORE (R. DE). See COMBEROUSSE (C. DE).

PASSANO (L. M.). Plane and spherical trigonometry. New York, Macmillan, 1918. 12mo. 16 + 141 pp. Cloth. $1.25

STRASSBURGER (A.) and CHANKIN (J.). Systematic drill in arithmetic. Books 2 and 3. New York, Longmans, 1918. 8vo. 10 + 180 + 10 + 181 pp. $0.45 + 0.45

III. APPLIED MATHEMATICS.

BECKER (K.). See CRANZ (C.).

BIGELOW (F. H.). A treatise on the sun's radiation and other solar phenomena. New York, Wiley, 1918. 9 + 385 pp. Cloth. $5.00

BOUASSE (H.). Optique géométrique élémentaire. Focométrie, optométrie. Paris, Delagrave, 1918. 8vo. 28 + 326 pp. Cartonné. Fr. 20.00

—— Optique géométrique supérieure. Paris, Delagrave, 1918. 8vo. 30 + 316 pp. Cartonné. Fr. 22.00

BOUSSINESQ (J.). Poussée des terres. Recherche des lois générales de l'état ébouleux produit dans un massif de sable par les déformations planes, parallèles à un plan vertical. Paris, Gauthier-Villars, 1917. 4to. 80 pp. Fr. 6.00

BRAUN (H.). Extra-premiën verblijf van verzekerden buiten Europa. Bekroond antwoord opeen door de Nationale Levensverzekering-Bank uitgeschreven prijsvraag. Rotterdam, Nationale Levensverzekering-Bank, 1915. Gr. 8vo. 130 pp.

COWLEY (W. L.) and LEVY (H.). Aeronautics in theory and experiment. London, Arnold, 1918. 11 + 284 pp. 16s.

CRANZ (C.). Lehrbuch der Ballistik. 1ter Band: Aeussere Ballistik oder Theorie der Bewegung des Geschosses von der Mündung der Waffe ab bis zum Eindringen in das Ziel. Unter Mitwirkung von K. Becker. 2te Auflage. Gleichzeitig 3te und vollständig umgearbeitete Auflage des "Compendiums der theoretischen äusseren Ballistik" von 1896. Leipzig, Teubner, 1917. Gr. 8vo. 16 + 528 pp. M. 19.00

CREHORE (A. C.). The mystery of matter and energy. New York, Van Nostrand, 1917. 11 + 161 pp. $1.00

EXTRAIT de la Connaissance des temps à l'usage des marins. Ephémerides nautiques pour l'an 1919, publiées par le Bureau des Longitudes. Paris, Gauthier-Villars, 1917. 8vo. Fr. 2.40

FÖPPL (A.). Vorlesungen über technische Mechanik. 1ter Band: Einführung in die Mechanik. 5te Auflage. Leipzig, Teubner, 1917. 8vo. 16 + 431 pp. M. 10.00

GOUARD (E.) et HIERNAUX (G.). Cours élémentaire de mécanique industrielle. Tome 1. 2e édition, revue, corrigée et augmentée. Nouveau tirage. Paris, Dunod et Pinat, 1918. 16mo. 8 + 386 pp. Cartonné. Fr. 6.00

GUILLOT (L.). Cours de mécanique. (Programme des écoles nationales d'arts et métiers.) Tome 4: Moteurs à gaz et à pétrole. Paris, Béranger, 1918. Cartonné. Fr. 19.25

HIERNAUX (G.). See GOUARD (E.).

LECORNU (L.). Cours de mécanique professé à l'Ecole polytechnique. Tome 3. Paris, Gauthier-Villars, 1918. 4 + 665 pp. Fr. 25.00

LEVY (H.). See COWLEY (W. L.).

LIVENS (G. H.). The theory of electricity. Cambridge, University Press, 1918. 8vo. 717 pp. 30s.

MALLIK (D. N.). Optical theories: based on lectures delivered before the Calcutta University. Cambridge, University Press, 1917. 6 + 181 pp. 7s. 6d.

MAREC (E.). La technique du croquis et du dessin industriel. Paris, Dunod et Pinat, 1917. 4to. 6 + 125 pp. Fr. 9.00

ROSE (W. N.). Mathematics for engineers. Part 1, including elementary and higher algebra, mensuration and graphs, and plane trigonometry. London, Chapman and Hall, 1918. 14 + 510 pp. 8s. 6d.

SOULEYRE (A.). La structure des planètes. Bône, Thomas, 1917. 8vo. 152 pp.

WARD (L. F.). Glimpses of the cosmos. Volume 6. Period 1897–1912. New York, Putnam, 1918. 34 + 410 pp. $1.75

YOUNG (A. S.). See YOUNG (C. A.).

YOUNG (C. A.). Lessons in astronomy. Revised by A. S. Young. Boston, Ginn, 1917.

TWENTY-SEVENTH ANNUAL LIST OF PAPERS.

READ BEFORE THE AMERICAN MATHEMATICAL SOCIETY AND SUBSEQUENTLY PUBLISHED, INCLUDING REFERENCES TO THE PLACES OF THEIR PUBLICATION.

BARROW, D. F. An application of Fourier's series to probability. Read Feb. 24, 1917. *Annals of Mathematics*, ser. 2, vol. 19, No. 2, pp. 96–105; Dec., 1917.

BEATTY, S. Derivation of the complementary theorem from the Riemann-Roch theorem. Read April 29, 1916. *American Journal of Mathematics*, vol. 39, No. 3, pp. 257–262; July, 1917.

BELL, E. T. Fourier series for the squares of Hermite's eighteen doubly periodic theta quotients. Read (San Francisco) April 6, 1918. *Messenger of Mathematics*, vol. 47, No. 4, pp. 49–57; Aug., 1917.

—— Numerical functions of $[x]$. Read (San Francisco) April 6, 1918. *Annals of Mathematics*, ser. 2, vol. 19, No. 3, pp. 210–216; March, 1918.

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BIRKHOFF, G. D. A theorem on series of orthogonal functions with an application to Sturm-Liouville series. Read Oct. 27, 1917. *Proceedings of the National Academy of Sciences*, vol. 3, No. 11, pp. 656–659; Nov., 1917.

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BLUMBERG, H. A theorem on semi-continuous functions. Read (Southwestern Section) Dec. 1, 1917. *Bulletin of the American Mathematical Society*, vol. 24, No. 8, pp. 381–383; May, 1918.

BRINK, R. W. A new integral test for the convergence and divergence of infinite series. Read April 29, 1916. *Transactions of the American Mathematical Society*, vol. 19, No. 2, pp. 186–204; April, 1918.

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—— Pierre Laurent Wantzel. Read Sept. 4, 1917. *Bulletin of the American Mathematical Society*, vol. 24, No. 7, pp. 339–347; April, 1918.

CAMP, B. H. Multiple integrals over infinite fields, and the Fourier multiple integral. Read Feb. 27, 1915, and April 29, 1916. *American Journal of Mathematics*, vol. 39, No. 3, pp. 311–334; July, 1917.

CARMICHAEL, R. D. Note on convergence tests applicable to series converging conditionally. Read (Chicago) April 6, 1917. *Tôhoku Mathematical Journal*, vol. 11, No. 4, pp. 191–199; June, 1917.

—— On the asymptotic character of functions defined by series of the form $\Sigma\, c_n\, g(x+n)$. Read (Chicago) April 21, 1916. *American Journal of Mathematics*, vol. 39, No. 4, pp. 385–403; Oct., 1917.

—— Elementary inequalities for the roots of an algebraic equation. Read Oct. 27, 1917. *Bulletin of the American Mathematical Society*, vol. 24, No. 6, pp. 286–296; March, 1918.

—— Comparison theorems for homogeneous linear differential equations of general order. Read (Chicago) April 6, 1917. *Annals of Mathematics*, ser. 2, vol. 19, No. 3, pp. 159–171; March, 1918.

—— On the representation of functions in series of the form $\Sigma\, c_n\, g(x+n)$. Read (Chicago) April 7, 1917. *American Journal of Mathematics*, vol. 40, No. 2, pp. 113–126; April, 1918.

CARPENTER, A. F. Some fundamental relations in the projective differential geometry of ruled surfaces. Read (Chicago) April 2, 1915, and (San Francisco) Nov. 25, 1916. *Annali di Matematica*, ser. 3, vol. 26, No. 4, pp. 285–300; Oct., 1917.

CHITTENDEN, E. W. On the equivalence of relations $K_{q_1 q_2 \infty}$. Read (Chicago) April 3, 1915. *American Journal of Mathematics*, vol. 39, No. 3, pp. 263–271; July, 1917.

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